PHYS 320 ANALYTICAL MECHANICS

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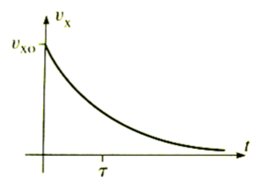
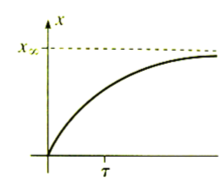
Fall 2018

Linear Air Resistance: $\vec{f}_{lin} = -b\vec{v}$

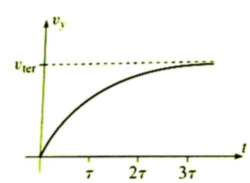
Allows horizontal solutions





$$v_x(t) = v_{xo}e^{-bt/m} = v_{xo}e^{-t/\tau}$$

$$x(t) = v_{xo}\tau(1-e^{-bt/m}) = v_{xo}\tau(1-e^{-t/\tau})$$



Allows vertical solutions

$$v_y(t) = \frac{mg}{b} + (v_{yo} - \frac{mg}{b})e^{-bt/m} = v_{ter} + (v_{yo} - v_{ter})e^{-t/\tau}$$

$$y(t) = v_{ter}t + (v_{yo} - v_{ter})\tau(1-e^{-t/\tau})$$

$$v_{ter} \equiv \frac{mg}{b} \qquad \tau \equiv \frac{m}{b}$$

Linear Air Resistance: projectiles

$$\begin{cases} x(t) = v_{xo}\tau\,(1-e^{-t/\tau}) \\ y(t) = v_{ter}t + (v_{yo} - v_{ter})\,\tau\,(1-e^{-t/\tau}) \end{cases}$$

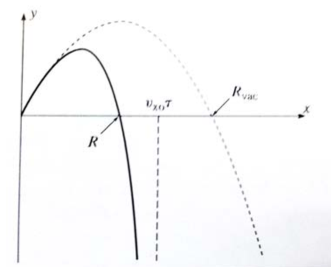
$$v_{ter} \equiv \frac{mg}{b} \qquad \tau \equiv \frac{m}{b}$$

$$\Rightarrow \quad y = \frac{v_{yo} + v_{ter}}{v_{xo}}x + v_{ter}\tau \ln\left(1 - \frac{x}{v_{xo}\tau}\right)$$

The range can be approximated (Taylor series) as

$$R = x(y=0) \approx \frac{2v_{xo}v_{yo}}{g}\left(1 - \frac{4v_{yo}}{3v_{ter}}\right)$$

(low air resistance)



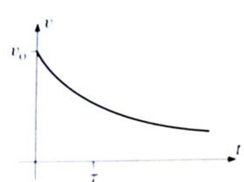
Quadratic Air Resistance:

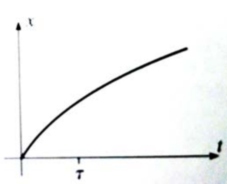
$$\vec{f}_{quad} = -c\,v\,\vec{v}$$

Allows horizontal solutions

$$v_x(t) = \frac{v_o}{1 + cv_o t/m} = \frac{v_o}{1 + t/\tau}$$

$$x(t) = v_o\tau \ln(1 + t/\tau)$$





Allows vertical solutions

$$v_y(t) = v_{ter}\tanh\left(\frac{gt}{v_{ter}}\right)$$

$$y(t) = \frac{v_{ter}^2}{g}\ln(\cosh\left(\frac{gt}{v_{ter}}\right))$$

$$v_{ter} \equiv \sqrt{\frac{mg}{c}} \qquad \tau \equiv \frac{m}{cv_o}$$